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# Budget Allocation for Vulnerable and Interdependent Civil Infrastructure Networks

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Abstract: Current literature on infrastructure management models water, sewer and road infrastructure into isolated layers and overlooks the spatial and functional interdependencies among these assets. Therefore, most decision support systems focused on the development of optimal strategies for rehabilitation and replacement of isolated networks. Vulnerability assessment can measure to which degree an asset's functionality can be compromised by the condition of another asset based on the expected spatial and functional interdependencies. The purpose of this paper is to provide a framework to allocate budgetary resources based on the recognized effect of spatial and functional interdependency of water, sewer and road assets on their likely vulnerability. The proposed computational platform consists of: 1) vulnerability and interdependency assessment model and 2) mitigation action model. The vulnerability and interdependency model is composed of two modules: 1) interdependency module and 2) vulnerability module. The interdependency module applies geospatial analysis in ArcGIS to recognize the interdependent layers of waters, sewers and roads and characteristics of such interdependencies. Subsequently, the vulnerability module rates to which extend the condition of a water, sewer or road asset can compromise the functionality of other assets. The vulnerability module utilizes fuzzy neural network to determine the vulnerability rating. Mitigation actions are assigned to each asset based on its vulnerability. Single objective genetic algorithm optimization is utilized to allocate budgetary resources to decrease likely vulnerability of assets being considered. Hypothetical water and sewer infrastructure network is used to demonstrate the application of the proposed framework and its expected contributions.

#### 1 Introduction

Asset management combines the economic and engineering principles with the intention of maintaining infrastructures to meet customers' preferences and avoid catastrophic failures. Generally, the asset engineers divide the civil infrastructure networks into isolated water, sewer and road networks. Subsequently, the performance and operation modeling of these systems is performed to support planning and maintenance from multiple view points, including infrastructure owners, investors, private and public users, and government entities. As such, the developed models focus on isolated analysis of infrastructure assets for specific domain (i.e. water, sewer, roadways...etc.) ignoring spatial and functional interdependencies. For instance consider a water pipeline attached to a bridge, a failure in that bridge will not affect only the adjacent road networks but the cascading consequences will progress to the water pipeline networks. Additionally, a failure of a water asset may drop the performance of adjacent water assets and may lead to water outages that affect the customers' satisfaction. Therefore, due to spatial and functional interdependencies, failure in one asset may not only affect the structural resilience and functionality of its network but may likely also compromise the functionality of other interdependent networks as well.

The objective of this paper is to provide a framework to allocate budgetary resources; accounting for the effect of spatial and functional interdependency of water, sewer and road assets on their likely vulnerability.

The paper starts by presenting the current state of art of interdependency and vulnerability assessment models and highlights some of the current limitations in the literature. Subsequently, the proposed research framework is presented to address some of the highlighted gaps in the literature. The implementation of the two models (i.e. vulnerability and interdependency assessment model, and mitigation action model) is presented in the framework implementation section. A case study using hypothetical water and sewer network is then presented to highlight the expected potentials for deploying such a framework. The paper concludes with the current limitation of the proposed research framework and outlines the expected future work.

## 2 Interdependency

Infrastructure systems interdependency is a growing area of study with contributions from multiple researchers in various engineering, mathematical and social science disciplines. It primarily focuses on aiding decision makers to achieve national security, economic prosperity, and the quality of life of today's societies (Gesara et al., 2010). Such economic and social prosperity is attained while heavily depending on the continuous and reliable operation of critical interdependent infrastructures. The study of interdependent infrastructures is challenging due to heterogeneous characteristics of infrastructure systems, insufficient data availability and the need to account for their spatial and functional aspects and its effect on supply-demand operation (Gesara et al., 2010).

# 2.1 Interdependencies Classifications

In the recent decade, researchers tried to understandand present classifications for infrastructure interdependencies that suites their domain of applications. Table 1 summarizes these classifications and their respective definitions.

Table 1 Interdependencies classifications

Classification	Definition	Rinaldi et al. (2001)	Lee et al. (2004)	Dudenhoeffer et al. (2006)
Physical	The physical output of one infrastructure is the physical input to another infrastructure.	Y	N	Υ
Cyber	Infrastructure is dependent on another infrastructure as they are connected via information links.	Υ	N	N
Geographical	Two infrastructures are dependent because of physical proximity.	Υ	N	Υ
Policy /procedural	Infrastructure state is dependent on another infrastructure state due to governmental policy or procedure.	N	N	Υ
Social	Infrastructure event may have influence on the community such as public opinion, public confidence and cultural issues.	N	N	Υ
Mutual	One infrastructure asset in an infrastructure group is depending on another infrastructures' functionalities	N	Y	N
Shared	The same infrastructure is utilized in providing two or more services to a group of infrastructures	N	Y	N
Exclusive	Two or more services can be provided by one infrastructure component at a time.	N	Y	N

<sup>\*</sup>considered: Y (Yes) - N (No)

In this paper, civil infrastructures interdependencies are limited to spatial and functional interdependencies between water, sewer and road assets. The spatial and functional interdependencies definitions are close to geospatial and physical interdependencies definitions suggested by other authors in Table 1 with some modifications to suite the integrated asset management framework. In this paper, spatial interdependencies addresses whether an infrastructure's structural resilience or performance is threaten by being located in the same geospatial area as another asset. Also, Functional interdependencies mean that an infrastructure's performance is limited by the structural resilience or performance of another asset.

# 2.2 Interdependency Models

Roughly, infrastructure interdependency modeling can be categorized into mathematical and simulation models. Mathematical models aim to abstract the network of these assets using graph theory and assess the interdependency using degree of vertices, average shortest path and clustering coefficient (Crucitti et al., 2003). Mathematical models are used to assess the degree of inoperability of asset networks due to imposed action (Haimes and Jiang, 2001, Santos and Haimes, 2004). On the other hand, the simulation models aim to simulate infrastructure networks through agent-based modeling (ABM) (Dudenhoeffer et al., 2006) or system dynamic simulation (SD) (Stapelberg, 2011) to encapsulate the interactions between various networks and users.

## 3 Vulnerability Assessment

Vulnerability can be defined in this paper as the extent to which an asset's performance is compromised due to performance interdependencies among the assets being considered. Hence, there are two types of vulnerabilities: 1) spatial vulnerability and 2) functional vulnerability. Spatial vulnerability represents the degree of susceptibility to structural failure as a result of being spatially interdependent with neighbouring assets. On the other hand, functional vulnerability represents the degree of susceptibility to functional failure as a result of being functionally interdependent with neighbouring assets. Vulnerability assessment is the process of identifying systems' weaknesses due to specific events and assessing the extent of such weaknesses on systems' performance or existence (Baker, 2003). Vulnerability assessment is carried out on two stages: 1) qualitative assessment which aims to identify the expected threats and 2) quantitative assessment which determines the likelihood and consequences for such identified threats on the system. Researchers tried to presents various methodologies for identifying system vulnerability which falls into two categories:

- 1- Objective scoring methods utilizing simple scoring method, multi-attribute utility theory and analytical hierarchy processes (Baker, 2003), Ezell, 2004), Karmakar et al., 2010).
- 2- Simulation methods utilizing agent based modeling or system dynamic simulation. (Eun et al (2010), Ouyang et al (2009)).

## 4 Limitations in Current Literature

The main limitations of the methods described above are:

- 1- Overlooking infrastructure interdependencies: Infrastructure Decision Support Systems (DSS) for rehabilitation, maintenance and mitigation actions are implemented ignoring the underlying spatial and functional interdependencies that exist between water, sewer and road networks (Moselhi et al, 2005).
- 2- Context and scope of interdependency models: Interdependency models are primarily concerned with the functional interdependency rather than the spatial interdependency for the domain of disaster management. In the context of disaster management, the decision maker should be able to restore the service in minimal time in order to cope with community expectation. (Baker, 2003; Dudenhoeffer et al., 2006).
- 3- Scale of modeling: The size of networks to be represented is a significant computational challenge. As the size of the modeled network increases, the scenarios' space to be

- investigated will necessary increases with complex, heterogeneous, interdependent infrastructure systems (Rinaldi et al., 2001; Moselhi et al., 2005). Vulnerability models are implemented mainly to study the vulnerability in one network, ignoring underlying interdependencies with other infrastructure networks (Ezell, 2004).
- 4- Research methods and data availability: Models are often limited by the amount and quality of information that infrastructure facility owners and managers are willing to share with public and private professional and academic entities, greatly reducing the generic applicability of the models and tools to be used in real-life scenarios (Earl et al., 2004).

#### 5 Research Framework

As mentioned previously, the objective of this paper is to provide a framework to allocate budgetary resources; accounting for the effect of spatial and functional interdependency of water, sewer and road assets on their likely vulnerability. The research methodology started with a comprehensive literature review that was carried out to cover three topics: 1) interdependency assessment, 2) vulnerability assessment, and 3) mitigation models. The literature review aimed at studying and understanding factors affecting interdependency and vulnerability and how can different interdependencies affect the vulnerability of civil infrastructures. It was carried out to identify research gaps (limitations) addressed in the previous section. Based on the review conducted, two models are proposed to achieve the above mentioned objective:

- 1- Vulnerability and interdependency model: This model can be divided into two modules:
  - Interdependency module: This model encapsulates the spatial and functional interdependency between three groups: 1) water and sewer, 2) water and road, and 3) sewer and road. For Spatial interdependency, the factors considered are buried depth, soil type, asset condition, asset alignment and asset criticality. On the other hand, the factors considered in the functional interdependency are pipe diameter, number of connected assets, customer types, and customer numbers. The module deploys ArcGIS 10<sup>TM</sup> capabilities to perform geospatial and functional interdependency analysis between water, sewer and road networks. Python is utilized to automate this process inside the ArcGIS 10<sup>TM</sup> and facilitate data exporting and importing to the vulnerability module.
  - II. Vulnerability module: The output of interdependency model will be used as an input to the vulnerability model that utilize fuzzy neural networks (FNN) to rate the vulnerability of the interdependent assets. FNN is a suitable technique when there is a lack of historical data and experts' interview can be used to overcome such limitation. A fuzzy-Neural tool in the Matlab TM will be used to perform the vulnerability module analysis.
- 2- Mitigation model: This model utilizes a number of mitigation actions to alleviate the impact of vulnerability on each asset's network. Mitigation actions in this model are limited to rehabilitation and replacement action to enhance the asset performance and decrease its likely vulnerability. The objective of the mitigation action model is to minimize vulnerability impact on asset networks within budget constraints. Single objective genetic algorithm is deployed to find near optimal solution. Infrastructure systems are considered as large scale systems that require near optimal solution that satisfies decision makers objectives in reasonable computational time. Hence, nonexact methods prevail when the objective is to find near optimal solutions over large space of water, sewer and road assets. The literature review covered a wide range of non-exact algorithms like tabu search, local search strategy, simulated annealing and genetic algorithms. Tabu search is a metaheuristic local search algorithm that can be used for solving combinatorial optimization problems (problems where an optimal ordering and selection of options is desired) (Talbi, 2009). In tabu search, an initial solution is randomly generated at each iteration and this solution is added to a list containing all the generated solutions to avoid replicating these solutions in the next iteration (Hillier and Lieberman, 2004). On the other hand, local search strategy starts with one solution and tries to improve over it for number of iterations and stops

when the next solution is worse than the previous. Simulated annealing is also single solution algorithm and the method is inspired by the physical process of heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy (Talbe, 2009). For each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance of the new point from the current point, or the extent of the search, is based on a probability distribution. The algorithm accepts all new points that lower the objective. but also, with a certain probability, points that raise the objective. By accepting points that raise the objective, the algorithm avoids being trapped in local minima in early iterations and is able to explore globally for better solutions (Talbi, 2009; Taha, 2008). Genetic Algorithms seem suitable as optimization technique over single population non exact algorithms like tabu-search, local search procedure, simulated annealing for a number of reasons (Talbi, 2009; Taha, 2008; Hillier and Lieberman, 2004). It allows for exploration of wide search space and genetic operators are utilized to improve characteristics of the search space (Talbi, 2009). The optimization algorithm is implemented inside the Matlab TM to facilitate the communication with the FNN model. The research framework is shown in Figure 1. The above models inputs, processes and outputs are explained in the framework implementation.

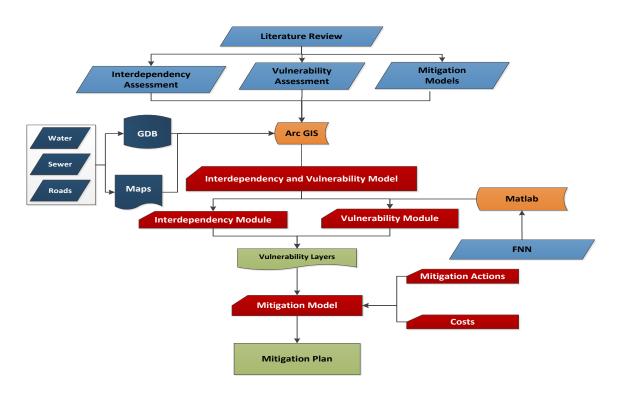


Figure 1: Research framework

## 6 Framework Implementation

## 6.1 Interdependency Module

The interdependency module is implemented to encapsulate:

- 1) The infrastructure assets that are spatially interdependent, co-located in the XYZ plans, using ArcGIS 10<sup>TM</sup> geoprocessing toolbox.
- 2) The infrastructure assets that are functionally interdependent using an algorithm that will trace the impact of an asset failure on the network functionality.

Geoprocessing is a methodical execution of a sequence of operations on geographic data to create new information using two process; spatial analysis and automation. This can be utilized by taking two different datasets (i.e. waters and roads) and find a new single dataset with the intersected assets and their corresponding attributes. For spatial interdependency, selection queries using location attributes is used to select the intersected layers of water, sewer and roads. Subsequently, union module of the geoprocessing toolbox is deployed to formulate three new layers of the intersected assets (waters and sewers - roads and sewers - roads and waters) as shown in Figure 2. The output is a new layer with new datasets that contain characteristics of the intersected assets. For instance, the new fields contain: which roads and sewers are intersected, the soil type between them, the distance between the two assets... etc. Hence, these factors can be used for the vulnerability module to assess the extent of any asset failure on spatially interdependent assets.

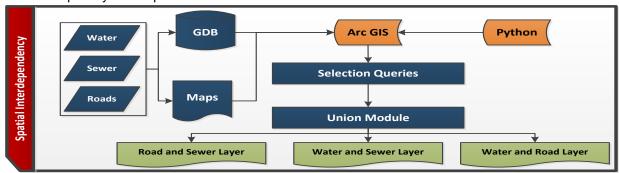


Figure 2: Spatial interdependency algorithm

Subsequently, the functional interdependency algorithm is utilized to determine to which extent an asset failure can affect other parts of the network. For example, the algorithm takes a water pipe and aggregates the number of pipes affected by such pipe failure, the number of affected customers, type of customers (commercial, industrial, domestic), as shown in Figure 3. These data are added to the geodatabase and are used in the functional vulnerability rating.

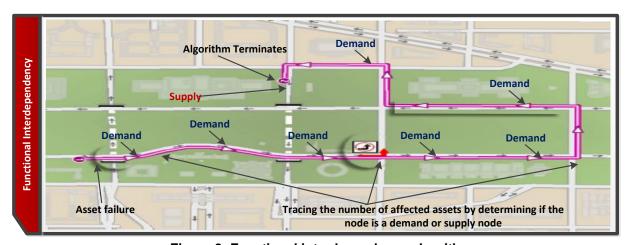


Figure 3: Functional interdependency algorithm

# 6.2 Vulnerability Module

A FNN will be utilized to rate the vulnerability of interdependent assets. FNN is suitable technique when there is a lack of historical data and interviews of experts can be used to overcome such limitation. In this model, questionnaires will be sent to experts and decision makers responsible for managing the water, sewer and road networks. These questionnaires will focus on three topics:

1. Eliciting the respondents about factors affecting spatial vulnerability of interdependent water, road and sewer networks. Respondents will be asked to determine the likely effect of factors like soil,

- buried depth, asset alignment on the spatial vulnerability between two assets. Also, respondents will be asked to state any factors that can be added to that list with justification and its expected effect on the vulnerability rating as well.
- 2. Eliciting the respondents about factors affecting functional vulnerability of interdependent water, road and sewer networks. Respondents will be asked to determine the likely effect of factors like customer type, customer number, and number of affected assets due to that asset failure on the functional vulnerability between the asset and its network. Also, respondents will be asked to state any factors that can be added to that list with justification and its expected effect on the vulnerability rating as well.
- 3. Respondents will be given hypothetical scenarios and in these hypothetical scenarios they will be asked to determine the vulnerability rating of two interdependent assets based on number of factors. These will be used as a vehicle to perform two tasks: 1) Training the FNN model to find the best membership function that represents the considered factors and its effect on vulnerability. 2) Testing the model to verify and validate the generated output to overcome lack of historical data on the interaction between the three systems.

The output of the vulnerability module is displayed in three layers, representing the vulnerability rating of each water, road and sewer asset. The detailed process of vulnerability module is shown in Figure 5. The questionnaire will target a wide range of experts in the water, sewer and roads infrastructure management sector to formulate an expert system that can be adopted into the current context of asset management. Therefore, the data collection and implementation of this model is still ongoing process.

## 6.3 Mitigation Action Model

For infrastructure asset management, the decision maker needs to implement a mitigation plan that decreases the losses over time while coping with budgetary constraints. The rationale in this model is to assign mitigation actions based on vulnerability of the assets and hence calculate total costs and corresponding decrease in vulnerability. Mitigation actions in this model are limited to rehabilitation and replacement actions for water, sewer and road networks. In any realistic situation, the costs due to such mitigation actions may exceed the budget therefore the decision maker will be directed to sub-optimal solutions. Genetic Algorithms are suitable for such modeling, where exact solutions may not be feasible, but finding near optimal solution over large solution space can be. In this model, single-objective genetic algorithm (SOGA) is used to minimize the vulnerability of the entire network. The proposed chromosome structure is shown in Figure 4.

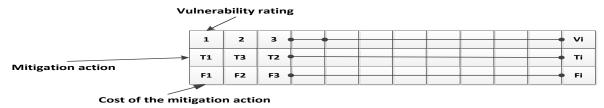


Figure 4: Chromosome structure

The objective function and constrains can be articulated as the following:

Minimize total vulnerability= $Min (\sum_{n=1}^{n} V_W + \sum_{j=1}^{j} V_S + \sum_{m=1}^{m} V_R)$	Equation 1
Subject to	
1. TB ≤ CB	Equation 2
$2.  TV \leq EV$	Equation 3

Where

 $V_W$ ,  $V_W$ ,  $V_R$  are vulnerability of water, sewer and roads networks respectively.

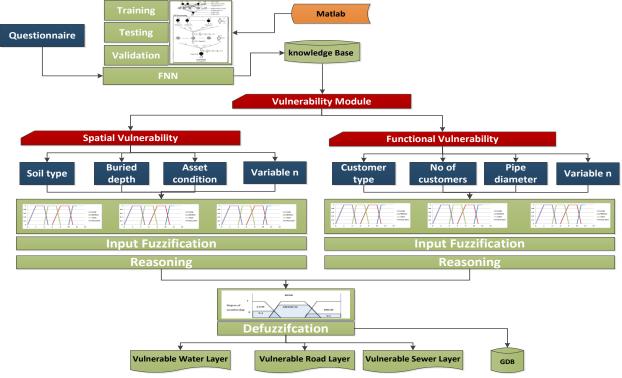


Figure 5: FNN model

TB and CB are the total budget and constrained budget respectively.

TV and EV are total vulnerability and expected vulnerability respectively.

The SOGA starts by generating random chromosomes (Talbi, 2009). Each chromosome has a number of genes where each gene represents the excepted vulnerability rating for that asset with the associated type of mitigation and its cost. The selection of mitigation action is done randomly. Each chromosome represents intervention plan that is evaluated via fitness function shown in Equation 1. After evaluating all the possible plans and if the objective function is not fulfilled, chromosomes are selected from the initial population to be parents for reproduction. Selection is made based on fitness and done randomly. Crossover is a recombination operator that proceeds in three steps: The reproduction operator selects randomly a pair of two individual strings for the mating; then a crossover point is selected randomly along the string length, and finally, the position values are swapped between the two strings following the crossover point. After crossover, the strings are subjected to mutation. Mutation prevents the algorithm to be trapped in a local minimum and plays the role of recovering the best genetic materials because of randomly disturbing genetic information by crossover. These steps are repeated on the offspring generation till the optimum solution is encountered. The SOGA is implemented using Matlab

## 7 Case Study

The hypothetical case study presented in this section illustrates the functionality of the mitigation action model. In this hypothetical model, 28 water and sewer pipeline are considered with various vulnerability rating and are divided into 7 zones as shown in Figure 6. The mitigation model utilizes GAs for the optimization of the mitigation plan for the water and sewer network. Three actions were considered in that model: 1) do nothing (D), 2) minor rehabilitation (IR) and 3) major rehabilitation (MR). The termination condition (available budget for the mitigation) for water and sewer assets was set at \$150,000. The following GA parameters were used: probability of mutation (0.2), probability of crossover (0.6), and

population size (500). The module analysis was performed on a Quad core 1.9 GHz processor and with 8 Gigabytes of memory. The module output is shown in Figure 7. The total cost for the mitigation is \$130,255, which is below the constrained budget by 13.2 %. Comparing Figure 6 and Figure 7, the mitigation model arrived at relaxed plan that assign MR to high vulnerable assets, IR to medium vulnerable assets and D to low vulnerable assets in most cases.

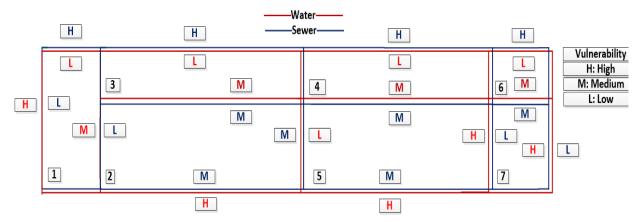


Figure 6: Water and sewer network

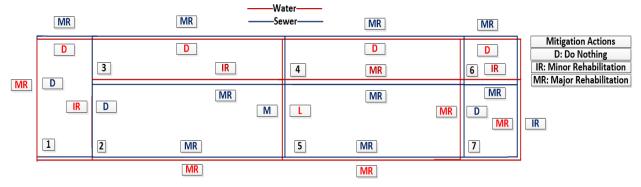


Figure 7: Mitigation actions

## 8 Summary and Conclusion

The paper presented a framework to allocate budgetary resources, while accounting for the effect of spatial and functional interdependency of water, sewer and road assets on their likely vulnerability. The proposed computational platform consists of: 1) vulnerability and interdependency assessment model and 2) mitigation action model. The interdependency model identifies the spatial and functional interdependencies by the interdependency module. The vulnerability module rates to which extend water, sewer or road asset can compromise the functionality of other assets utilizing fuzzy neural networks. The mitigation actions are assigned to each asset based on the expected vulnerability using SOGA optimization. Hypothetical water and sewer infrastructure networks were used to demonstrate the application of the proposed framework and its essential features. The vulnerability model data collection and implementation is ongoing process and it's still underdevelopment. Also, the mitigation action model needs further expansion to consider multi-objective optimization to account for various preferences of the decision makers (e.g. minimize vulnerability, minimize the total budget, maximize the network performance, etc). Furthermore, the mitigation action model should be linked to a simulation model to understand how a certain mitigation action can affect the performance of the infrastructure and hence customers' expectations for better rationale of the budget allocation process.

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