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A PROPOSED APPROACH FOR CONSTRUCTION PROJECT COST RISK ANALYSIS USING INFLUENCE DIAGRAMS

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Abstract: Construction project cost risk analysis is a challenging process from both the owner's and contractor's viewpoints due to the uncertainty in construction projects and often the lack of adequate historical data on risks. In the construction industry, a variety of methods, including Monte Carlo simulation, have traditionally been used for estimating project cost risk. However, such methods typically rely on the presence of adequate historical data and have limitations in considering important aspects of risk analysis, including (1) determining, modelling, and analyzing the root causes of risks, (2) considering interrelationships and correlations among risk factors, (3) accounting for over-counts of risk factors (i.e., considering the effect of a risk factor more than once due to the lack of transparency in the risk assessment process), and (4) incorporating both expert knowledge and historical data. In this paper, an approach to cost risk analysis using influence diagrams is suggested due to their effectiveness in addressing the complexities of cost risk analysis of construction projects. Fuzzy logic is suggested for use in handling the subjective uncertainty regarding expert knowledge, a source of information for risk assessment. A framework is presented for the quantitative evaluation of cost risks based on influence diagrams, integrated with a hybrid Monte Carlo method that handles both probabilistic and fuzzy inputs.

1. Cost Risk Analysis in Construction

1.1 Introduction

The actual total cost of a project is usually different from the initial estimate as a variety of factors can influence the cost of a project's work packages. Therefore, there is a need to account for risks that can result in variability of estimated costs, and to plan for appropriate response strategies to minimize the risk of over-expenditure over the course of project. Construction projects are unique and complex in nature, which makes it difficult to obtain adequate historical data for statistical risk analysis. The accuracy of cost risk analysis can be crucial in different phases such as the project appraisal, budgeting, contingency allowance determination, and determining appropriate risk response strategies. The competitive advantage of construction companies also relates to the accuracy of risk analysis, since an overestimated cost risk results in an overestimation of contingency (included in the project budget), and therefore a loss of opportunity. On the other hand, underestimating cost risk level can also impact companies, as cost overruns are very common in the construction industry.

The severity of a risk event has traditionally been assessed based on two attributes: probability of occurrence and magnitude of impact. Therefore, risk assessment is basically a quantification and aggregation of these two attributes of risk. What is important to consider with regard to quantification of these attributes is the available sources of information and the type of uncertainty. Two sources of data can be used as the basis for risk quantification—namely, historical data and expert knowledge. Using appropriate methods of data elicitation and quantification to deal with the available and reliable data sources is critical to achieving an accurate risk analysis result. Moreover, depending on the source of data, different types of uncertainty are involved in risk analysis—each of which can be better handled by different techniques. Handling both attributes of risk (i.e., probability and magnitude of impact) involves a number of complexities, some of which are explained in the next section. The success of risk analysis practices depends on their ability to handle these complexities.

1.2 Complexities of Cost Risk Analysis in Construction

1.2.1 Availability and Reliability of Data

Construction projects, far from being repetitive outputs of a factory line that are manufactured under almost the same circumstances, are each unique. Due to this uniqueness, objective data with which to assess the probability of events' occurrences are typically insufficient, and a high degree of subjective assessments are required (Flanagan and Norman 1993; Tah et al. 1993). Inadequate objective data in many cases implies that statistical analysis can be impossible or unreliable, and using expert knowledge is necessary. However, when sufficient historical data are available for some factors and the risk factors are not project-dependent, statistical analysis is of value and should not be ignored; thus, a system that is capable of using both sources of data can be helpful in making effective use of available data.

1.2.2 Root Cause Analysis and Transparency of the Risk Model

One of the most important phases of cost risk analysis is root cause analysis. Root cause analysis involves identifying the causes of cost risks and determining the degree to which these causes influence the variability of costs. By using influence diagrams, root cause analysis can begin from the cost item level, and continue by finding the root causes of risk factors until reaching the point of finding risk factors at the project level. Root cause analysis is of such importance that some authors such as Hollmann (2007) have argued that a contingency estimate "is not an end in itself; it is part of a driver-focused process" and must be based on the quantified influences of risk factors. However, modelling the cost risk based on its root causes is complex, and some of the very common practices such as line-by-line Monte Carlo simulation do not consider the root causes. One shortcoming of such methods is their inability to consider the interrelationships among the variables of the risk model in the absence of adequate historical data for the purpose of statistical correlation analysis.

When the root causes (risk factors) of variables in a risk model are not transparently modelled, some risk factors can be over-counted in different steps of estimation and risk analysis. For example, the effect of weather on a cost item may be accounted for once in the base estimate by the estimator, only to be considered again in other phases of risk analysis.

Modelling the risk factors of cost items is a complex process that requires a systematic and structured approach. Influence diagrams are graphical tools that can facilitate the process of risk factor identification and modelling by visually illustrating the root causes of cost items, and using graphical properties such as nodes and arcs to determine the root causes of risk factors and the interrelationships (correlations) among them. Influence diagrams also reduce the probability of over-counting, as they provide a transparent structure of risk factors.

1.2.3 Handling Uncertainty

The term "risk" has been defined differently by different authors, but one aspect that is common among the definitions is their emphasis on the uncertainty of an event in terms of probability of occurrence and magnitude of impact. What is complex in risk assessment is effective quantification and analysis of uncertainty. The probability of occurrence of a risk event, and the degree to which the event can affect the total project cost are two important uncertainties regarding project cost risk. Moreover, using expert knowledge as an information source, there is a degree of imprecision in linguistic assessments that introduces yet another aspect of uncertainty into the input data. Before discussing the methodology of handling these aspects of uncertainty in project cost risk analysis, two types of uncertainty regarding the probability of an event must be investigated and clarified: objective uncertainty and subjective uncertainty.

Flanagan and Norman (1993) introduce two schools of thought with regard to probability of event occurrence. Probabilities relate frequencies of occurrence over a long period of time. In other words, repeatability and trial and/or observations are what can be used in considering probability. On the other hand, subjective probability refers to "the degree of belief or confidence" one places in the occurrence of an event based on information available to him/her. Flanagan and Norman also argue that subjective probabilities are generally used in construction industry assessments, as decision makers have to make decisions based on experience. These authors suggested that all available objective and subjective data should be used in assigning subjective probability that reflects the decision maker's opinion.

Fuzzy probability introduced by Zadeh (1984) is a method to obtain a degree of belief or certainty, and is therefore a suitable method of handling subjective uncertainty. Zadeh argued that the probability regarding events with subjective and imprecise descriptions can be better expressed as possibility distributions. The probability of drawing a large ball out of an urn with balls of different sizes is an example Zadeh used to describe such types of probabilities.

One aspect of uncertainty that arises when using expert knowledge is the ambiguity and vagueness of linguistic data. Input data from experts is often expressed in linguistic format. Using linguistic inputs increases the level of human centricity in the risk analysis process and, according to Schumucker (1976), also increases the level of accuracy of input data. Schmucker reports the results of a psychological study that shows 16% to 32% higher level of consistency in responses when verbal expression is provided compared to using a precise answer on a subjective concept.

Fuzzy probability can be a better option than probability theory for handling probabilities of uncertain events that also have ambiguity in their definitions (such as probability of occurrence of a *high* temperature). However, if adequate and reliable historical data on a parameter of a risk model exist, the use of probability distributions is preferred to the use of fuzzy numbers. As Guyonnet et al. (2003) argue, "Compared with a probability density function, a fuzzy number is 'poor' in terms of intrinsic information. A probability density function defines a variable entirely, but applies best to systems which are 'closed' such as the throw of a dice. But the systems of interest in an environmental context are 'open' and possibility theory may appear as an eligible alternative for representing some of the uncertainty pertaining to such systems".

In cost risk analysis of construction projects, adequate historical data can be obtained for some risk factors such as material unit prices, trades' hourly wages, and equipment cost rate. Therefore, this paper suggests the use of historical data to handle the uncertainty of these types of risk factors, referred to henceforth as "internal risk factors".

2. Using Influence Diagrams for Project Cost Risk Analysis

Influence diagrams are graphical tools that can facilitate the process of risk factor identification and modelling by graphically illustrating the root causes of cost items and their interrelationships (correlations). Because of their transparent structure, influence diagrams also reduce the probability that factors will be over-counted.

According to Yu et al. (1994), an influence diagram is "an acyclic directed graph in which the variables of the decision model are represented as chance nodes and the probabilistic dependencies among variables are represented as directed arcs among nodes". Diekmann et al. (1996) describe influence diagrams as cost risk analysis tools that can conveniently be used to model the relationships between risks and costs; the authors further describe influence diagrams as being especially advantageous when considering the external risks that are not taken into account during the estimation phase, particularly when these risks can be described by the use of conditional probabilities.

In this paper, the categorization of risk factors by Diekmann et al. (1996) is used to distinguish between two types of risk factors: internal and external. Internal risk factors, such as labour and equipment hourly rates and material unit prices, are used directly in calculating the base estimates of cost items. External risk factors, such as geotechnical conditions or material supply delay, also influence the cost items; however, they are not included in the calculation of the base estimates. This distinction is important when considering the different data sources that will be used to handle the uncertainty involved in assessing risk factors.

As discussed previously, using influence diagrams in risk analysis facilitates risk factor modelling, root cause analysis, and increased transparency in the risk analysis process, while handling the problem of correlations. However, a quantitative analysis of the influence of risk factors on each other and on the overall project cost is a complexity of this approach that must be overcome. This issue has previously been addressed by taking a number of different approaches. According to Diekmann et al. (1996), the majority of these approaches are based on conditional (Bayesian) probability theory. In this paper, a different approach is proposed: fuzzy numbers are used to indicate the probability of external risk factors, and probability distribution functions are used for internal risk factors. This approach is based on the available sources of information—that is, expert knowledge for the external factors, and historical data for the internal ones. Fuzzy set theory is more suited to handling the uncertainty involved in the subjective assessments made by experts and the vagueness involved in their linguistic assessments, while probability theory is more suited to handling the uncertainty of factors with historical data. Probability distribution functions can be obtained based on the historical data and a time series to transform the historical data into probability distribution functions.

The structure of an influence diagram includes nodes that represent risk factors in this application, and arcs that show the influence of factors on other factors. In some approaches, such as the one being proposed in this paper, weights are also used to show the degree of influence of one factor on another. Three types of nodes may be used in an influence diagram: chance nodes, decision nodes, and value nodes. A chance node is used for a random variable; a decision node indicates that a decision is to be made; and a value (utility) node maps the permutation of the predecessor nodes to a single value or utility. The relationships among the nodes are illustrated through the use of arcs that indicate conditional or informational relationships (Shachter 1986). Section 3 provides an example of an influence diagram that is designed for cost risk analysis of a single cost item (steel reinforcement). Influence diagrams for cost risk analysis typically consist of chance nodes and a value node at the end; however, in case a decision node is required to consider possible scenarios, the diagram can be evaluated for one possible scenario at a time and the results can then be compared. Three basic problems that should be solved in order to evaluate an influence diagram are discussed in the following sections.

2.1 Propagation

When multiple nodes are connected and each successor node has only one predecessor, calculating the cumulative effect of predecessors of one node is referred to in this paper as "propagation". If P_i denotes the probability of risk factor (node) i, and E_{ij} denotes the effect of node i on node$j$, then calculating E_{ij} based on P_i and the influence weight of i on j (W_{ij}) is "propagation". In the existing literature of influence diagram quantitative evaluation, propagation through the nodes and arcs of an influence diagram is usually handled by the use of conditional (Bayesian) probabilities. In this approach, the probability of an event (i.e., risk factor or node in the case of influence diagrams), given that another event has already occurred (parent node), is calculated based on Bayes' theorem of conditional probability. The result can be viewed as an If-Then rule explaining the propagation in an influence diagram. Another approach is to determine the If-Then rules using experts' direct input. Then, a rule-based system is typically used to handle the calculations regarding the determined rules and input data. This approach requires experts' input for every possible scenario considered in the influence diagram, which, for large and complex influence diagrams, would be difficult and time-consuming for experts to assess.

Both of these approaches can be also applied to an influence diagram with fuzzy inputs. Operations on the fuzzy inputs to determine the propagation computation of an influence diagram are not restricted to the Bayesian approach. In this paper, a class of conjunctive fuzzy aggregators—triangular norms (t-norms)—are utilized to handle the propagation computation for fuzzy influence diagrams designed for cost risk analysis.

2.2 Compound Aggregation

In a situation where multiple arcs are connected as inputs of a node—as in Figure 1, where nodes 6 and 7 influence node 4 simultaneously—the aggregated result of the predecessor nodes on one node is referred to as "compound aggregation" in this paper. Similar to the propagation problem, different approaches can be taken to solve the compound aggregation problem. For example, compound probabilities can be calculated through the use of joint distributions. Alternatively, If-Then rules can be directly obtained from experts to conduct the compound aggregation. In this paper, using triangular conorms (t-conorms or s-norms) as a class of disjunctive operators is suggested. The advantage of using this approach, as opposed to collecting rules from experts, is that it does not require experts' input for all the compound aggregation instances so it is more time efficient. Additionally, a large class of operators can be used, providing flexibility by allowing the aggregator operators to be replaced by other operators or to be tuned based on actual results.

2.3 Aggregation of Fuzzy and Probabilistic Data

Internal risk factors are defined as those directly involved in estimating a cost item. These risk factors are basically the unit prices of required material and the labour and equipment rates. In contrast to external risk factors, which are usually project dependent and require expert evaluation of their riskiness, internal risk factors have attainable historical data that are reliable and independent from the project conditions. For example, the chance of shortage in skilled labour for a certain task depends on many project-specific factors. However, an internal risk factor such as steel unit price does not depend on the characteristics of a particular project, as it is influenced by market conditions and other economic factors.

Economic models can be developed to forecast material unit prices and equipment and labour rates; however, these models are very complex and require detailed market and economy research that is beyond the scope of construction project risk analysis. However, since historical data on these internal risk factors are attainable, a statistical analysis can be conducted to forecast the most probable cost and the variability of these factors. In order to consider the variability of cost of these factors, Monte Carlo simulation, which requires probability distributions as input data, can be used. For Monte Carlo simulation, time series methods that produce probability distribution functions from historical data can be implemented.

Once the probability distributions regarding the internal risk factors are obtained, they can be used as inputs to the risk model. External factors, on the other hand, can be quantified by the use of fuzzy sets, which are effective in handling the uncertainty of the linguistic terms describing such factors. The result of propagation and compound aggregations of fuzzy sets are fuzzy sets; therefore, a hybrid aggregator is needed to be able to use both fuzzy and probabilistic data and aggregate them into a single result. In the next section of this paper, a hybrid Monte Carlo simulation method will be employed to simulate the variability of internal risk factors and to handle their aggregation with external risk factors.

3. Proposed Method of Cost Risk Analysis and Illustrative Example

In order to benefit from the advantages of using influence diagrams in root cause analysis and correlation analysis, a quantitative method of evaluating an influence diagram is required. In the previous section, the major problems of evaluating an influence diagram were briefly reviewed, and in this section a method that handles the stated problems is proposed. A simplified example of a fuzzy influence diagram illustrating the risk of a cost item (i.e., steel reinforcement) has been provided to explain the proposed method. The influence diagram is evaluated through the use of triangular norm and triangular conorm fuzzy aggregation operators; t-norms and s-norms are two classes of fuzzy operators that handle the set operations and logical inference of fuzzy sets.

Consider Figure 1 as an example of an influence diagram for cost risk analysis of a steel reinforcement operation as a cost item. The influence diagram considers immediate components (internal risks) of cost estimation for the cost item (i.e., rebar unit price and labour wage per hour). A variation in the quantities of these factors with regard to the cost item, which are denoted as Q_1 and Q_2 for rebar and labour hours respectively, influence the cost item. Next, external risk factors that can influence the required quantity of rebar and labour hours are considered. The next step is to identify and model the root causes of the influencing risk factors, and then continue the same process for the identified risk factors until reaching the risk factors at the project level. For example, once the internal factors affecting steel reinforcement (i.e., rebar unit price and labour hourly wage) are determined, risk factors that can affect the required quantities of these factors can be determined and a root cause analysis of these factors conducted. In this example, a possible scope change and/or schedule change are recognized as potential risk factors (external factors) that may cause a variation in the required quantity of rebar and/or required labour hours for the cost item.

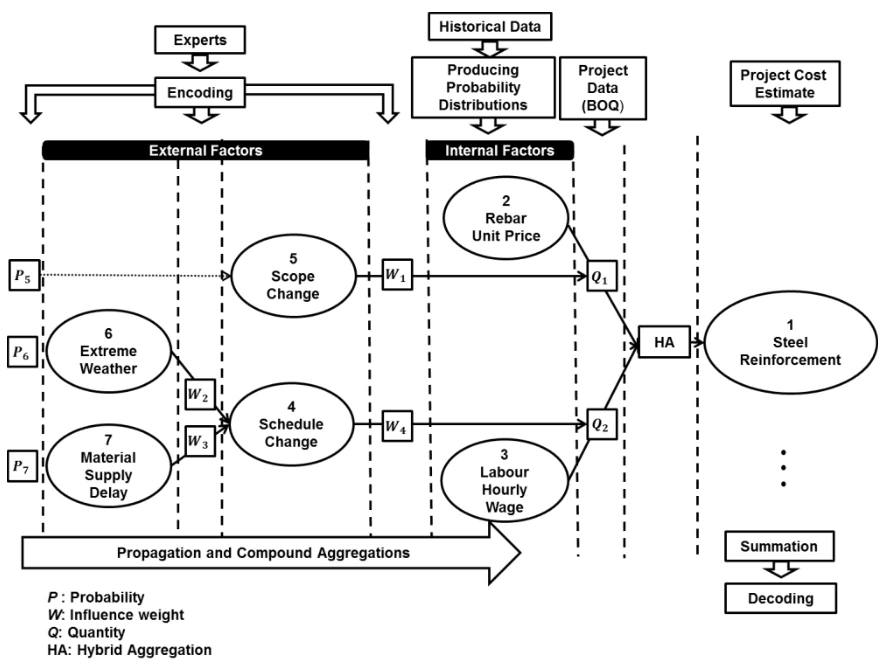

Figure 1: An illustrative example of an influence diagram, information sources, and evaluation process

For simplicity's sake in the example, root cause analysis of external factors is restricted to the root causes of schedule change (i.e., extreme weather and material supply delay) and is not continued for scope change. Root cause analysis to determine the factors that can change the rebar unit price and labour hourly wage is not considered in this approach, as it is beyond the realm of construction risk analysis and is a subject better suited to economics and market research. Our approach suggests the use of historical data, which are easily obtained, for assessment of the internal risk factors. Input data for the internal risk factors are probability distributions of the factors such as rebar unit price and labour hourly wage, and the quantity of these factors from the bill of quantities or other project documents (i.e., Q_1 and Q_2 in this example). The weight of external risk factors' influence on other factors (W_i) should be determined by experts. For those external factors that establish the parent nodes, the probability of occurrence should also be provided by experts (P_i). The probabilities of intermediate nodes are determined through the fuzzy causal operations (propagation and compound aggregation). A hybrid Monte Carlo method will combine probabilistic and fuzzy data and provide a fuzzy set that indicates the total risk for the cost item.

In order to collect experts' assessments for external risk factors (P_i and W_i), it is necessary to determine a number of choices of linguistic terms, referred to as 'frames of cognition' by Pedrycz and Gomide (2007). Next, a fuzzy number should be assigned to each linguistic term using encoding (fuzzification) techniques, so as to develop fuzzy sets for each linguistic variable expressed by experts. Once an expert assesses a variable by using one of the linguistic terms, the associated fuzzy number is used as the quantified input to the system. The sources of input data and general operations are shown in Figure 1. In order to calculate the variability risk of a cost item based on the previously discussed structure of an influence diagram, the following steps are proposed.

Step 1: Aggregate Effect of External Risk Factors into a Single Fuzzy Set
Beginning with the external factors, use Equation 1 to calculate the effect of parent nodes on their child nodes (E_{ij}s). Then, calculate the probability of each child node ($\mathrm{P_i}$ s) based on E_{ij}s. If a node has just one predecessor, its P_j is equal to its only E_{ij}; otherwise, the compound aggregation of E_{ij}s should be calculated by the procedure explained later in this section. Once the probability of each child node has been assigned, the parent nodes can be removed from the influence diagram and the deducted influence diagram can be viewed as a new influence diagram. In this example, schedule change is influenced by its parents: extreme weather and material supply delay (i.e., external risk factors). Therefore, by evaluating the effect of external risks and calculating the probability of schedule risk, the external risks can be removed from the influence diagram.

Propagation. The effect of each risk factor on another risk factor can be calculated using a t-norm as a conjunction (AND) logical connection. For example, one can say, "The probability of extreme weather conditions is high AND extreme weather has a high influence on the construction production rate so the severity of the risk of a schedule change is very high". Equation 1 indicates this computation where t denotes a t-norm.

[1] $E_{ij} = P_i \;\mathrm{t}\; W_{ij}$

The choice of t-norm can influence the accuracy of this computation. Therefore, using expert input and/or case study data to validate the result of different t-norms applied to some hypothetical examples can be useful in increasing the accuracy of the evaluation of risk factor effects. The product operator can be used as a t-norm (t_p) to calculate E_{ij}s, similarly to how the product operator has traditionally been used to calculate weighted factors.

Compound Aggregation. When a risk factor, such as schedule change in the example, has more than one parent, a disjunctive aggregation applies, which can be handled by the use of a s-norm, such as a maximum s-norm, Lukasiewicz s-norm, or probabilistic sum s-norm. As in propagation, a suitable choice of s-norm can be very effective in improving the accuracy of the result; the choice of s-norm can be validated using expert knowledge and/or case study data. Considering $i = \{1,2, \dots, n\}$ as parent nodes of node j, Equation 2 indicates a disjunctive aggregation using a s-norm(s).

[2] $E_j = [E_1 \;\mathrm{s}\; E_2 \;\mathrm{s} \dots\; E_n]$

With regard to the illustrative example shown in Figure 1, after applying Step 1 the resulting influence diagram will be a reduced influence diagram as depicted in Figure 2. Here, A^*denotes the aggregated effect of the removed part of the original influence diagram. $A^*$is calculated as in Equation 3.

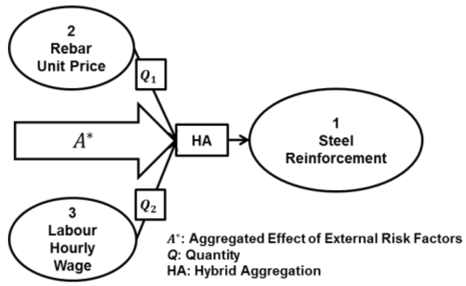

Figure 2: The influence diagram for the illustrative example after conducting Step 1

[3] $A^* = \left[\left((P_6\ t\ W_2) s\ (P_7\ t\ W_3) \right) t\ W_4 \right] s\ [P_5\ t\ W_1]$

Step 2: Apply hybrid Monte Carlo simulation

After completing Step 1, all the external risk factors are aggregated into a fuzzy set. This fuzzy set indicates the total level of risk from external factors that directly influence the cost item. Internal risk factors that are obtained from historical data and established probability distributions remain. Based on the definition of internal risk factors, the estimation of the cost item is a function of the internal factors and their required quantities for the cost item. For example, using the example, Equation 4 is the estimated cost of steel reinforcement.

[4] $Steel\ Reinforcement\ Cost = Rebar\ Unit\ Price\ \times\ Q_1 + Labour\ Hourly\ Wage\ \times\ Q_2$

Hybrid Aggregation. In this method, the uncertainty regarding the internal risks is considered using probabilistic distributions. On the other hand, the uncertainty regarding the external risk factors is considered by the use of fuzzy sets, which are aggregated into a single fuzzy set A^* resulting from Step 1. Therefore, a hybrid method is required to handle both probabilistic and fuzzy data. Previous practices of introducing such a hybrid method include using possibility/probability transformation (Wonneberger et al. 1995), using belief functions (Shafer 1976), using α-cutsand interval computation together with random sampling and Monte Carlo simulation (Guyonnet et al. 2003), and using fuzzy Monte Carlo simulation involving fuzzy arithmetic and the development of fuzzy cumulative distribution functions (CDFs) (Sadeghi et al. 2010).

A method that is commonly used to effectively handle probabilistic uncertainty is Monte Carlo simulation. Guyonnet et al.'s (2003) hybrid Monte Carlo method is used in this paper to handle both probability and fuzzy inputs, and to provide an aggregated output. A schematic illustration of this method, within the context of the proposed risk analysis method, is provided in Figure 3.

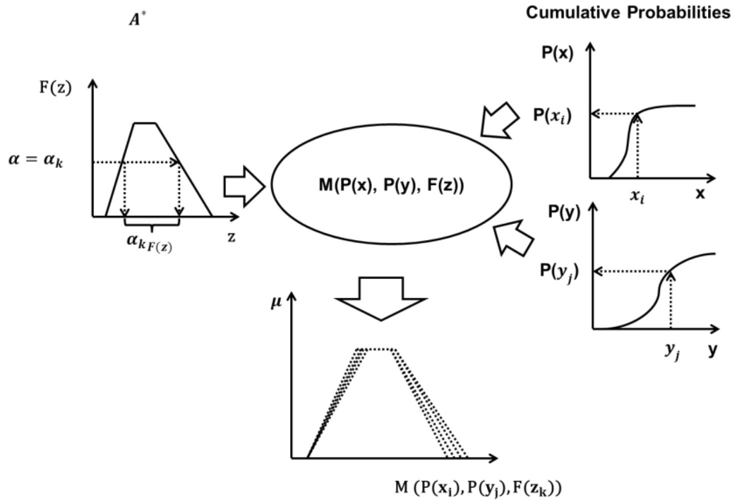

Figure 3: A representation of Guyonnet et al.'s (2003) hybrid Monte Carlo method

In Guyonnet et al.'s method, *n* sample numbers are first generated to obtain *n* values from the CDFs and to sample *n* probabilities. Then, an α membership level is selected, and, considering all the interval members and the sampled probabilities, the infimum and supremum of the model (M) is calculated. This procedure is repeated for several α levels. Using the representation theorem, the fuzzy output of the model can then be determined. The representation theorem states that a fuzzy set can be reconstructed by taking the union of its α -cuts. By ω repetitions of this process, ω fuzzy numbers will be generated. At each α level, a histogram of the cumulative infimum and supremum frequencies is generated to determine the boundaries of the α -cut at a certain probability level.

For the influence diagram model, only one fuzzy set results from the aggregation of all the external risk factors, making this process even simpler. However, since this aggregated fuzzy set is not necessarily convex as are the fuzzy numbers in Guyonnet et al.'s method, it is necessary to address this non-convexity. In such a situation, some α -cuts can be the union of two or more intervals that result from calculating the infimum and supremum values. This process can also be automated by including a line in the algorithm to exclude points between the infimum and supremum of an α -cut that are not members of the α -cut interval.

The model function (M) in the proposed influence diagram method is a conjunction of the aggregated external risks' effects (A^*) and the randomly sampled values of internal risks. Thus, the model function can be obtained by a t-norm such as the product operator. Since A^* implies the probability, and its universe of discourse is [0 1], the result of the conjunction of $A^*$and the base estimate of the cost item can be viewed as the expected value of the cost item. Equation 5 shows the model function of the illustrative example.

[5] $M(P_2, P_3, A^*) = A^*\ t\ (P_2 \times Q_1 + P_3 \times Q_2)$

The expected value of the total project cost is the summation of the fuzzy sets for all cost items, resulting from the two steps described in this section, which yields a fuzzy set. This resulting fuzzy set can be used to provide the possibility of different values for the expected total project cost, or it can be defuzzified to provide a crisp expected value of the total project cost.

4. Conclusion and Future Research

This paper reviews some of the complexities of cost risk analysis for construction projects, and suggests using influence diagrams for this purpose due to their advantages over other methods in root cause analysis, correlation analysis, and transparency of the resulting risk model. Furthermore, a framework for developing influence diagrams for cost risk analysis has been provided, and a quantitative method of evaluating influence diagrams with fuzzy and probabilistic inputs has been proposed. In the proposed method, fuzzy operators are used to handle the computation of the aggregated effect of multiple risk factors; in this way, experts' input is limited to the initial assessment of probabilities and influence degrees (weights). In addition, a hybrid Monte Carlo method is employed to consider the variability of risk factors with adequate historical data and to aggregate fuzzy and probabilistic inputs to a single fuzzy set output (i.e., the expected value of the total project cost).

In order to improve the quality of the results from the proposed method, different choices of t-norms and t-conorms (s-norms) will be explored and their accuracy compared in different contexts. Further research will consider alternative methods of aggregating fuzzy and probabilistic inputs, and will compare them to the one proposed in this paper in terms of reliability and applicability. Improving the quality and accuracy of data elicited from experts is another aspect of future research. Finally, the proposed approach will be validated by collecting expert knowledge and/or actual project data and applying it to a number of case studies, and comparing its results to those achieved by other existing methods of cost risk analysis.

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